< ODE : Ordinary differential Equations >

$$\frac{d}{dt} y(t) = f(t, y(t)), \quad y(t_0) = y_0$$

< Example >

$$\frac{dy}{dt} = -y - 5e^{-t} \sin 5t, \quad y(0) = 1$$

$\Rightarrow$ Exact sol $y = e^{-t} \cos 5t$.

< ode45 in Matlab >

$$[\underbrace{t}_{t_i}, \underbrace{y}_{y_i}] = \text{ode45}(@\underbrace{myf}_{\substack{f(t,y) \\ \text{defined ft}}}, \underbrace{tspan}_{\text{interval}}, \underbrace{yzero}_{\text{initial value}})$$

% myode1.m

% $\frac{dy}{dt} = -y - 5e^{-t} \sin 5t$, $y(0) = 1$

```
tspan = [0, 3];     yzero = 1;
[t, y] = ode45(@myf, tspan, yzero);
plot(t, y, '*--')
xlabel t,   ylabel y(t)

    function yprime = myf(t,y)
        yprime = -y - 5*exp(-t)*sin(5*t);
    end

max(abs(y - exp(-t).*cos(5*t)))
```

< Dynamical system >

(Example 1)

$$\begin{cases} y_1'(t) = y_2(t) \\ y_2'(t) = -\sin y_1(t). \end{cases}$$

(I.C) $(y_1(0), y_2(0)) = (1, 1)$ or $(-5, 2)$ or $(5, -2)$.

```
% ode_sys2.m
pend = @(t, y) [y(2); -sin(y(1))];
tspan = [0, 10];
yza = [1 ; 1]; yzb=[-5; 2]; yzc = [5; -2]
[ta, ya] = ode45(pend, tspan, yza);
[tb, yb] = ode45(pend, tspan, yzb);
[tc, yc] = ode45(pend, tspan, yzc);

plot(ya(:,1), ya(:,2), yb(:,1), yb(:,2), yc(:,1), yc(:,2))
xlabel y_1(t), ylabel y_2(t)
```

(Example 2) Rössler system

$$\begin{cases} y_1' = -y_2 - y_3 \\ y_2' = y_1 + a y_2 \\ y_3 = b + y_3 (y_1 - c) \end{cases}$$

a, b, c : parameters

$y_1(0) = y_2(0) = y_3(0) = 1.$

```
% ode_sys3.m
rossler = @(t,y,a,b,c) [-y(2)-y(3); y(1)+a*y(2); b+y(3)*(y(1)-c)];
tspan = [0, 100]; yz = [1;1;1];
options = odeset('AbsTol',1e-7,'RelTol',1e-4);
a = 0.2; b= 0.2; c = 2.5; %or c= 5
[t, y] = ode45(rossler, tspan, yz, options, a, b, c)
subplot(121); plot3(y(:,1),y(:,2),y(:,3)); grid
   title('c = 2.5'); xlabel y_1, ylabel y_2, zlabel y_3
subplot(122); plot(y(:,1), y(:,2))
   title('c = 2.5'); xlabel y_1, ylabel y_2
```

< The 2nd-order Initial value Problem >

$$\begin{cases} \theta''(t) + \sin\theta(t) = 0, \\ \theta(0) = 1, \quad \theta'(0) = 1 \end{cases}$$

Let $y_1(t) = \theta(t)$ and $y_2(t) = \theta'(t)$.

Then,
$$\begin{cases} y_1'(t) = \theta'(t) = y_2(t) \\ y_2'(t) = \theta''(t) = -\sin\theta(t) = -\sin y_1(t) \end{cases}$$

And $y_1(0) = \theta(0) = 1$, $y_2(0) = \theta'(0) = 1$.

System of IVP :

$$\begin{cases} y_1' = y_2 \\ y_2' = -\sin y_1 \end{cases} \qquad y_1(0) = 1, \quad y_2(0) = 1.$$

We can solve the system using Example 1.

< Pursuit Problem : rabbit and fox >

- Rabbit follows a predefined path $(r_1(t), r_2(t))$
- Fox chases the rabbit in such a way
  - a) at each moment the tangent of the fox's path points towards the rabbit
  - b) the speed of the fox is some constant k times the speed of the rabbit.

Then, the path $(y_1(t), y_2(t))$ of the fox is given by

$$\begin{cases} y_1'(t) = s(t)\,(r_1(t) - y_1(t)) \\ y_2'(t) = s(t)\,(r_2(t) - y_2(t)) \end{cases}$$

where $s(t) = \dfrac{k\sqrt{(r_1')^2 + (r_2')^2}}{\sqrt{(r_1 - y_1)^2 + (r_2 - y_2)^2}}$

Let $\begin{pmatrix} r_1(t) \\ r_2(t) \end{pmatrix} = \sqrt{1+t} \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$

and $y_1(0) = 3$, $y_2(0) = 0$.

```
% fox1.m : a function file
function yp = fox1(t,y)
 k = 0.75;   r = sqrt(1+t)* [cos(t); sin(t)];
 r_p = (0.5/sqrt(1+t))*[cos(t)-2*(1+t)* sin(t); ...
                 sin(t)+2*(1+t)*cos(t)];
 dist = norm(r-y);
 if dist > 1e-4
      st = k*norm(r_p)/dist;
      yp = st*(r-y)
 else
      error('ODE model ill-defined');
 end
```

< Stiff Problems : Robertson ODE System >

$$\begin{cases} y_1' = -0.04y_1 + 10^4 y_2 y_3 \\ y_2' = 0.04y_1 - 10^4 y_2 y_3 - 3 \cdot 10^7 y_2^2 \\ y_3' = 3 \cdot 10^7 y_2^2 \end{cases}$$

$[y_1(0), y_2(0), y_3(0)] = [1; 0; 0]; \quad 0 \leq t \leq 3.$

① Use ode45

② Use ode15s : Stiff Problem $\Big\}$ Implicit

or ode23s , ode23tb